

Zooming into Kinetic Equations using the Characteristic Mapping Method

Philipp Krah¹, Xi-Yuan Yin², Julius Bergmann¹, Jean-Christophe Nave³, and Kai Schneider¹

¹*I2M - Institut de Mathématiques de Marseille, Aix-Marseille Université, France*

²*LMFA, Ecole Centrale de Lyon, Université de Lyon, France*

³*McGill University, Montreal, Canada*

Our work introduces an efficient semi-Lagrangian characteristic mapping method (CMM) for solving kinetic equations with high precision on coarse grids. Its ability is demonstrated for the one+one-dimensional Vlasov–Poisson equation. This method employs a numerically evolved flow map utilizing a gradient-augmented level-set technique, achieving exponential resolution within linear time. We demonstrate the method’s global third-order convergence in space and time while rigorously evaluating its conservation properties.

Benchmarking against established test cases such as Landau damping and the two-stream instability showcases the effectiveness of our approach, highlighting its superiority over classical pseudo-spectral methods. Moreover, we illustrate the method’s exceptional capability to transcend the limits of currently available schemes by showcasing its extreme fine-scale resolution features, shown in fig. 1. Our method stands as a promising advancement in addressing the computational challenges associated with kinetic theories, offering a pathway towards unprecedented precision in simulations.

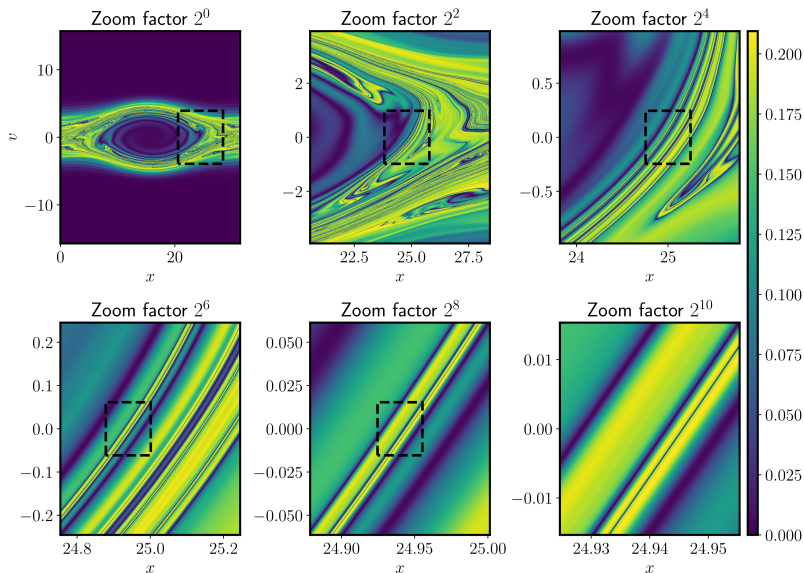


Figure 1: Two-stream instability computed with CMM. Successive zooms of the distribution function f at $t = 80$ to illustrate the multitude of resolved scales. The region inside the dotted black square is successively amplified by a factor 4 in each direction up to a factor of 2^{10} .

Acknowledgement: The authors were granted access to the HPC resources of IDRIS under the allocation No. AD012A01664R1 attributed by Grand Équipement National de Calcul Intensif (GENCI). Centre de Calcul Intensif d’Aix-Marseille is acknowledged for granting access to its high performance computing resources. The authors acknowledge partial funding from the Agence Nationale de la Recherche (ANR), project CM2E, grant ANR-20-CE46-0010-01.