Hierarchical Model Order Reduction for Rarefied Gases using Neural Networks

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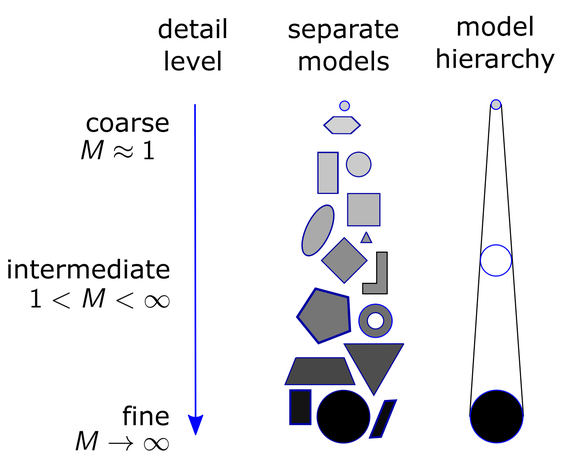
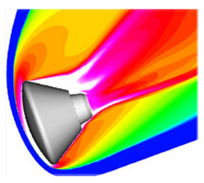
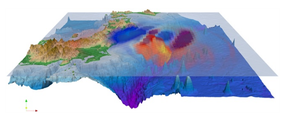
Moment Models and Model Hierarchy



a) rarefied gases



b) free-surface flows

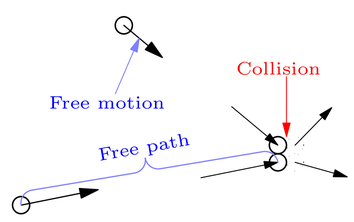


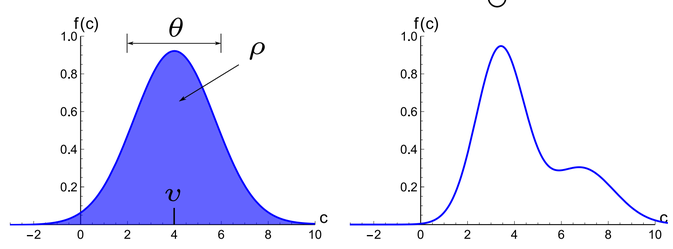
Fundamental Question: Which model do we need?

Kinetic Equations

Boltzmann Transport Equation

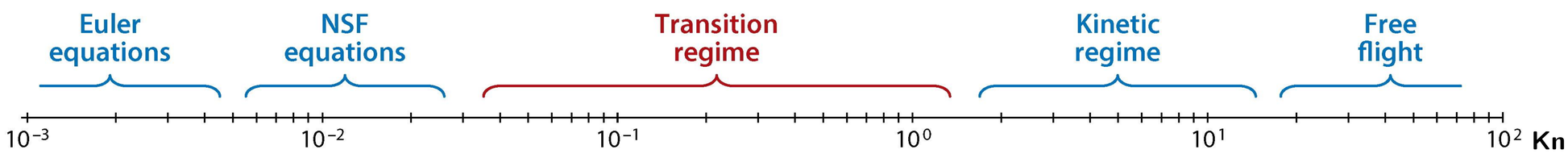
$$\frac{\partial}{\partial t} f(t, \boldsymbol{x}, \boldsymbol{c}) + c_i \frac{\partial}{\partial x_i} f(t, \boldsymbol{x}, \boldsymbol{c}) = S(f)$$





Euler equations → moment equations, M?

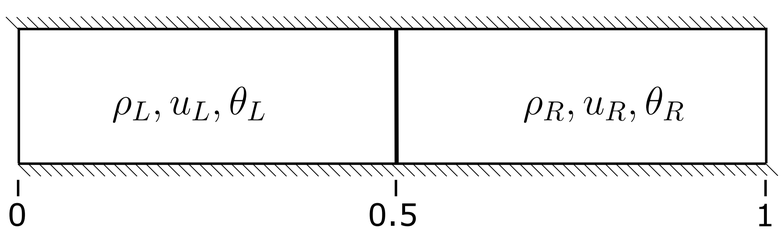
Models



Numerical Test Case: Sod Shock Tube

Discrete velocity method reference data: $f(t^n, x_i, c_j) \in \mathbb{R}^{N_t \times N_x \times N_c}$

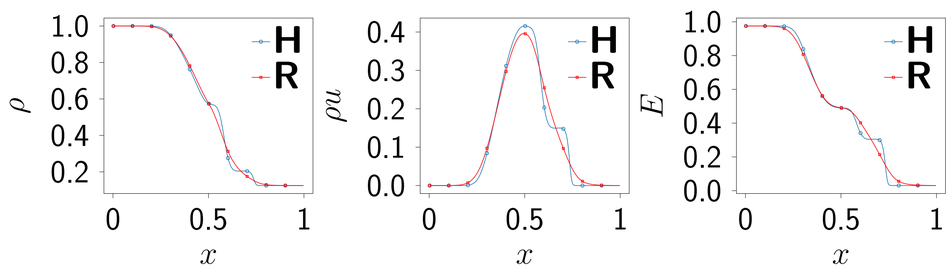
- domain $(x, c) \in [0, 1] \times [-10, 10]$
- $N_x = 200$ spatial points, $N_c = 40$ discrete velocities
- $N_t = 25$ time steps $t \in [0, 0.12]$



Rarefied and Continuum Flow Regime

Two regimes are computed using BGK equation:

- Rarefied flow / rarefied regime **R**: $\mathrm{Kn} = 10^{-2}$
- continuum flow / hydrodynamic regime **H**: $\mathrm{Kn} = 10^{-5}$



Methods: POD vs. Neural Autoencoder Networks

Proper Orthogonal Decomposition (POD)/Singular Value Decomposition (SVD)

- POD-Ansatz $f(t, x, c) = \sum_{i=1}^{n} \phi_i(x, t) a_i(c)$
- Method of Snapshots:

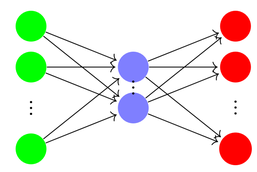
$$F = \begin{array}{c} c \\ \boxed{} \\ t \times x \end{array}$$

- truncated SVD $F \approx \tilde{F} = U_r \Sigma_r V_r^T$ with
  - $U_r \in \mathbb{R}^{(N_x N_t) \times r}$, $V_r \in \mathbb{R}^{(N_c) \times r}$ orthogonal $(V_r V_r^T = U_r^T U_r = 1)$
  - $\Sigma \in \mathbb{R}^{r \times r}$ containing singular values.

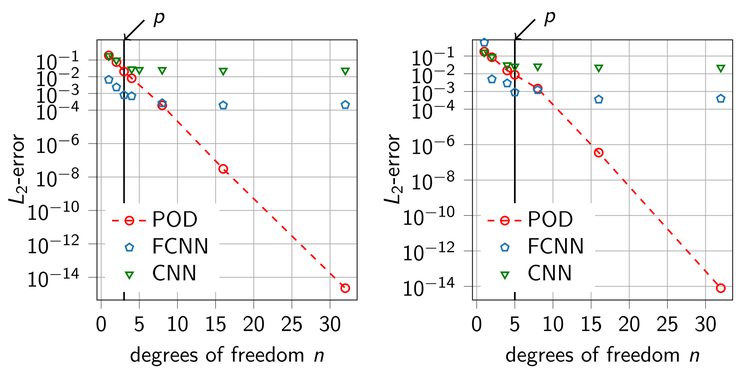
Neural Networks:

Two architectures used:

1. fully connected neural network (**FCNN**)
2. convolutional neural network (**CNN**)

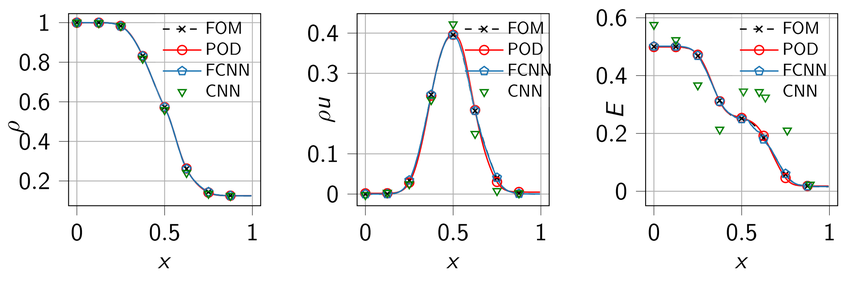


Results 1: How many moments?



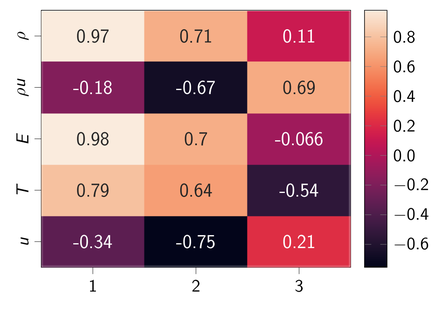
L_2 error with increasing n for **H** left and **R** right.

Results 2: Accuracy of macroscopic variables



Accurate macro solution of POD, FCNN, CNN (rarefied regime **R**).

Results 3: Which moments?



Pearson correlation between macroscopic and intrinsic variables.

References

[1] Z. Schellin, J. Koellermeier, P. Krah, Model Order Reduction for Rarefied Gases with POD and Neural Networks, *to be submitted*

[2] J. Koellermeier, H. Vandecasteele, Hierarchical Micro-Macro Acceleration for Moment Models of Kinetic Equations, *J. Comp. Phys.*, 488, 2023

[3] J. Koellermeier, P. Krah, J. Kusch. Macro-micro decomposition for consistent and conservative model order reduction of hyperbolic shallow water moment equations: A study using POD-Galerkin and DLRA, *submitted*